Conservation of Energy: The total sum of energy (in all its forms) is conserved in all interactions.

Conservation of Linear Momentum: In the absence of external torque, angular momentum is conserved in all interactions (vector relation).

Conservation of Charge: Electric charge is conserved in all interactions.

Conservation of Mass: (not valid)
MAXWELL’S EQUATIONS

- Gauss’s law for electricity: \( \oint E \cdot dA = \frac{q}{\varepsilon_0} \)
- Gauss’s law for magnetism: \( \oint B \cdot dA = 0 \)
- Faraday’s law: \( \oint E \cdot ds = -\frac{d\Phi_B}{dt} \)
- Generalized Ampere’s law: \( \oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I \)

LORENTZ FORCE LAW

Lorentz force law: \( \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \)

NEWTON’S LAWS

- **Newton’s first law**: Law of Inertia. An object in motion with a constant velocity will continue in motion unless acted upon by some net external force.
- **Newton’s second law**: The acceleration \( \mathbf{a} \) of a body is proportional to the net external force \( \mathbf{F} \) and inversely proportional to the mass \( m \) of the body. \( \mathbf{F} = m\mathbf{a} \)
- **Newton’s third law**: law of action and reaction. The force exerted by body 1 on body 2 is equal and opposite to the force that body 2 exerts on body 1.

LAWS OF THERMODYNAMICS

- **First law of thermodynamics**: The change in the internal energy \( \Delta U \) of a system is equal to the heat \( Q \) added to the system minus the work \( W \) done by the system.
- **Second law of thermodynamics**: It is not possible to convert heat completely into work without some other change taking place.
- **Third law of thermodynamics**: It is not possible to achieve an absolute zero temperature.
- **Zeroth law of thermodynamics**: If two thermal systems are in thermodynamic equilibrium with a third system, they are in equilibrium with each other.

FUNDAMENTAL FORCES

<table>
<thead>
<tr>
<th>FORCE</th>
<th>RELATIVE STRENGTH</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>1</td>
<td>Short, ( \sim 10^{15} ) m</td>
</tr>
<tr>
<td>Electroweak</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>10^{-2}</td>
<td>Long, ( 1/r^2 )</td>
</tr>
<tr>
<td>Weak</td>
<td>10^{-9}</td>
<td>Short, ( \sim 10^{-15} ) m</td>
</tr>
<tr>
<td>Gravitational</td>
<td>10^{-39}</td>
<td>Long, ( 1/r^2 )</td>
</tr>
</tbody>
</table>

ATOMIC MASS

The mass of an atom is its atomic number divided by the product of 1000 times Avogadro's number.

KINETIC ENERGY

The kinetic energy of a particle (ideal gas) in equilibrium with its surroundings is:

\[ K = \frac{3kT}{2} \]

PHASE SPACE

A six-dimensional pseudospace populated by particles described by six position and velocity parameters:

position: \((x, y, z)\) velocity: \((v_x, v_y, v_z)\)
RELATIVITY

<table>
<thead>
<tr>
<th>WAVELENGTH $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \lambda \nu$</td>
</tr>
<tr>
<td>$1\text{Å} = 10^{-10}\text{m}$</td>
</tr>
</tbody>
</table>

$c =$ speed of light $2.998 \times 10^8 \text{m/s}$  
$\lambda =$ wavelength [m]  
$\nu =$ (nu) radiation frequency [Hz]  
$\text{Å} =$ (angstrom) unit of wavelength equal to $10^{-10}\text{m}$  
$m =$ (meters)

Michelson-Morley Experiment indicated that light was not influenced by the “flow of ether”.

<table>
<thead>
<tr>
<th>RELATIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu \lambda = c$</td>
</tr>
<tr>
<td>$1\text{Å} = 10^{-10}\text{m}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME DILATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given two systems moving at great speed relative to each other; the time interval between two events occurring at the same location as measured within the same system is the proper time and is shorter than the time interval as measured outside the system.</td>
</tr>
<tr>
<td>$T' = \frac{T_0}{\sqrt{1 - v^2 / c^2}}$ or $T = \frac{T_0}{\sqrt{1 - v^2 / c^2}}$</td>
</tr>
</tbody>
</table>
| $T_0, T_0 =$ the proper time (shorter). [s]  
$T, T' =$ time measured in the other system [m]  
$v =$ velocity of $(x',y',z')$ system along the $x$-axis. [m/s]  
$c =$ speed of light $2.998 \times 10^8 \text{m/s}$ |

<table>
<thead>
<tr>
<th>LORENTZ TRANSFORMATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compares position and time in two coordinate systems moving with respect to each other along axis $x$.</td>
</tr>
</tbody>
</table>
| $x' = \frac{x - vt}{\sqrt{1 - v^2 / c^2}}$  
$t' = \frac{t - vx / c^2}{\sqrt{1 - v^2 / c^2}}$ |
| $v =$ velocity of $(x',y',z')$ system along the $x$-axis. [m/s]  
$t =$ time [s]  
$c =$ speed of light $2.998 \times 10^8 \text{m/s}$ |
| or with $\beta = \frac{v}{c}$ and $\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$  
so that $x' = \gamma(x - vt)$ and $t' = \gamma(t - \beta x / c)$ |

<table>
<thead>
<tr>
<th>LENGTH CONTRACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given an object moving with great speed, the distance traveled as seen by a stationary observer is $L_0$ and the distance seen by the object is $L'$, which is contracted.</td>
</tr>
<tr>
<td>$L_0 = \frac{L'}{\sqrt{1 - v^2 / c^2}}$</td>
</tr>
</tbody>
</table>
| $L_0 =$ the proper length (longer). [m]  
$L' =$ contracted length [m]  
$v =$ velocity of $(x',y',z')$ system along the $x$-axis. [m/s]  
$c =$ speed of light $2.998 \times 10^8 \text{m/s}$ |

<table>
<thead>
<tr>
<th>LIGHT WAVEFRONT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position of the wavefront of a light source located at the origin, also called the spacetime distance.</td>
</tr>
<tr>
<td>$x^2 + y^2 + z^2 = c^2 t^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RELATIVISTIC VELOCITY ADDITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Where frame $K'$ moves along the $x$-axis of $K$ with velocity $v$, and an object moves along the $x$-axis with velocity $u$, with respect of $K'$, the velocity of the object with respect to $K$ is $u$.</td>
</tr>
</tbody>
</table>
| $K$  
$u'$  
$v$  
$K'$ |
| $u = \frac{u' + v}{1 + (v/c^2)u'}$  
$u = \frac{u' - v}{1 - (v/c^2)u'}$ |
| $u_x =$ velocity of an object in the $x$ direction [m/s]  
$v =$ velocity of $(x',y',z')$ system along the $x$-axis. [m/s]  
$c =$ speed of light $2.998 \times 10^8 \text{m/s}$  
$\gamma = 1 / \sqrt{1 - v^2 / c^2}$ |
| If there is $u_y$ or $u_z$ within the $K'$ frame then  
$u_y = \gamma \left[ \frac{1 - (v/c^2)u_x} {1 - (v/c^2)u_x} \right]$  
and $u_z = \gamma \left[ \frac{u_x} {1 - (v/c^2)u_x} \right]$ |
| $u_y =$ velocity of $u_y$ in the $y$ direction [m/s]  
$v =$ velocity of $(x',y',z')$ system along the $x$-axis. [m/s]  
$c =$ speed of light $2.998 \times 10^8 \text{m/s}$  
$\gamma = 1 / \sqrt{1 - v^2 / c^2}$ |
For the situation where the velocity $u$ with respect to the $K$ frame is known, the relation may be rewritten exchanging the primes and changing the sign of $v$. |
**SPACETIME DIAGRAM**

The diagram is a means of representing events in two systems. The horizontal \( x \) axis represents distance in the \( K \) system and the vertical \( ct \) axis represents time multiplied by the speed of light so that it is in units of distance as well. A point on the diagram represents an event in terms of its location in the \( x \) direction and the time it takes place. So points that are equidistant from the \( x \) axis represent simultaneous events.

\[ ct' \]
\[ v = c \]

A system \( K' \) traveling in the \( x \) direction at \( \frac{1}{4} \) the speed of light is represented by the line \( ct' \) in this example, and is called a worldline. The line represents travel from one location to another over a period of time. The slope of the line is proportional to the velocity. A line with a slope of 1 (dashed line in illustration) indicates travel at the speed of light, so no worldline can have a slope less than 1. A straight line indicates zero acceleration. Simultaneous events occurring at \( t = t' = 0 \) in the \( K' \) system may be represented by points along the \( x' \) axis. Other simultaneous events in the \( K' \) system will be found on lines parallel to the \( x' \) axis.

\[ x' \]
\[ \text{slope} = \frac{v}{c} = \beta = 0.25 \]

\[ \text{Worldline} \]
\[ v = 0.25c \]
\[ \text{slope} = \frac{c}{v} = \beta = 4 \]

**SPACETIME INTERVAL \( \Delta s \)**

The quantity \( \Delta s^2 \) is invariant between two frames of reference with relative movement along the \( x \)-axis.

\[ s^2 = x^2 - (ct)^2 = x'^2 - (ct')^2 \]

Two events occurring at different times and locations in the \( K \)-frame may be characterized by their \( \Delta s^2 \) quantity.

\[ \Delta s^2 = \Delta x^2 - (c\Delta t)^2 \]

- **lightlike - \( \Delta s^2 = 0 \):** In this case, \( \Delta x^2 = c^2\Delta t^2 \), and the two events can only be connected by a light signal.

- **spacelike - \( \Delta s^2 > 0 \):** In this case, \( \Delta x^2 > c^2\Delta t^2 \), and there exists a \( K' \)-frame in which the two events occur simultaneously but at different locations.

- **timelike - \( \Delta s^2 < 0 \):** In this case, \( \Delta x^2 < c^2\Delta t^2 \), and there exists a \( K' \)-frame in which the two events occur at the same position but at different times. Events can be causally connected.

**MOMENTUM \( p \)**

\[ p = m v \quad \text{for a photon: } p = \frac{h\nu}{c} \]

- \( p \) = momentum [kg-m/s], convertible to [eV/c] by multiplying by \( c/q \).
- \( m \) = mass of the object in motion [kg]
- \( v \) = velocity of object [m/s]
- \( \nu \) = (nu) the frequency of photon light [Hz]
- \( c \) = speed of light \( 2.998 \times 10^8 \) m/s

**RELATIVISTIC MOMENTUM \( p \)**

\[ p = \frac{\gamma m u}{\gamma} \quad \text{where:} \]

- \( p \) = relativistic momentum [kg-m/s], convertible to [eV/c] by multiplying by \( c/q \).
- \( \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \)
- \( m \) = mass [kg]
- \( u \) = velocity of object [m/s]
DOPPLER EFFECT

Given two systems approaching each other at velocity \( v \), light emitted by one system at frequency \( v_0 \) (nu, proper) will be perceived at the higher frequency of \( v \) (nu) in the other system.

\[
\frac{v}{\sqrt{1 - \beta}} \quad \text{For two systems receding from each other, reverse the signs.}
\]

\( v = (\text{nu}) \) the frequency of emitted light as perceived in the other system [Hz]
\( v_0 = (\text{nu}) \) the proper frequency of the emitted light (lower for approaching systems) [Hz]. Frequency is related to wavelength by \( c = \lambda v \).
\( \beta = v/c \) where \( v \) is the closing velocity of the systems (Use a negative number for diverging systems.) and \( c \) is the speed of light \( 2.998 \times 10^8 \text{ m/s} \)
\( v \) = velocity of \((x',y',z')\) system along the \( x \)-axis. [m/s]

RELATIVISTIC KINETIC ENERGY \( K \)

Relativistic kinetic energy is the total energy minus the rest energy. When the textbook speaks of a 50 Mev particle, it is talking about the particle’s kinetic energy.

\[
K = \gamma mc^2 - mc^2
\]

where:
\( K \) = relativistic kinetic energy [J], convertible to [eV] by dividing by \( q \).
\( \gamma = 1/\sqrt{1 - v^2/c^2} \)
\( m \) = mass [kg]
\( c \) = speed of light \( 2.998 \times 10^8 \text{ m/s} \)

REST ENERGY \( E_0 \)

Rest energy is the energy an object has due to its mass.

\[ E_0 = mc^2 \]

TOTAL ENERGY \( E \)

Total energy is the kinetic energy plus the rest energy. When the textbook speaks of a 50 Mev particle, it is talking about the particle’s kinetic energy.

\[
E = K + E_0 \quad \text{or} \quad E = \gamma mc^2
\]

where:
\( E \) = total energy [J], convertible to [eV] by dividing by \( q \).
\( K \) = kinetic energy [J], convertible to [eV] by dividing by \( q \).
\( E_0 \) = rest energy [J], convertible to [eV] by dividing by \( q \).
\( \gamma = 1/\sqrt{1 - v^2/c^2} \)
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\( E_0 \) = rest energy [J], convertible to [eV] by dividing by \( q \).
\( \gamma = 1/\sqrt{1 - v^2/c^2} \)
\( m \) = mass [kg]
\( c \) = speed of light \( 2.998 \times 10^8 \text{ m/s} \)

MOMENTUM-ENERGY RELATION

\[
(\text{energy})^2 = (\text{kinetic energy})^2 + (\text{rest energy})^2
\]

\[
E^2 = p^2c^2 + mc^4
\]

where:
\( E \) = total energy (Kinetic + Rest energies) [J]
\( p \) = momentum [kg-m/s]
\( m \) = mass [kg]
\( c \) = speed of light \( 2.998 \times 10^8 \text{ m/s} \)

BINDING ENERGY

- the potential energy associated with holding a system together, such as the coulomb force between a hydrogen proton and its electron
- the difference between the rest energies of the individual particles of a system and the rest energy of the bound system
- the work required to pull particles out of a bound system into free particles at rest.

\[
E_B = \sum_i m_i c^2 - M_{\text{bound system}} c^2
\]

for hydrogen and single-electron ions, the binding energy of the electron in the ground state is

\[
E_B = \frac{mZ^2e^4}{2\hbar^2 (4\pi\varepsilon_0)^2}
\]

\( E_B \) = binding energy (can be negative or positive) [J]
\( m \) = mass [kg]
\( Z \) = atomic number of the element
\( e \) = \( q \) = electron charge [c]
\( \hbar \) = Planck’s constant divided by \( 2\pi \) [J-s]
\( \varepsilon_0 \) = permittivity of free space \( 8.85 \times 10^{-12} \text{ F/m} \)
\( c \) = speed of light \( 2.998 \times 10^8 \text{ m/s} \)

LINE SPECTRA

Light passing through a diffraction grating with thousands of ruling lines per centimeter is diffracted by an angle \( \theta \).

\[
d \sin \theta = n \lambda
\]

The equation also applies to Young’s double slit experiment, where for every integer \( n \), there is a lighting maxima. The off-center distance of the maxima is

\[
y = l \tan \theta
\]

\( d \) = distance between rulings [m]
\( \theta \) = angle of diffraction [degrees]
\( n \) = the order number (integer)
\( \lambda \) = wavelength [m]
\( l \) = distance from slits to screen [m]
WIEN'S CONSTANT  

The product of the wavelength of peak intensity $\lambda$ [m] and the temperature $T$ [K] of a blackbody. A \textit{blackbody} is an ideal device that absorbs all radiation falling on it.

$$\lambda_{\text{max}} T = 2.898\times10^{-3} \text{ m} \cdot \text{K}$$  

STEFAN-BOLTZMANN LAW  

May be applied to a \textit{blackbody} or any material for which the emissivity is known.

$$R(T) = \varepsilon \sigma T^4$$  

where:

$R(T) = \text{power per unit area radiated at temperature } T$ \hspace{1cm} [W/m$^2$]

$\varepsilon = \text{emissivity } (\varepsilon = 1 \text{ for ideal blackbody})$

$\sigma = \text{constant } 5.6705 \times 10^{-8} \text{ W/(m}^2\cdot\text{K}^4)$

$T = \text{temperature } (\text{K})$

PHOTON  

A photon is a massless particle that travels at the speed of light. A photon is generated when an electron moves to a lower energy state (orbit).

**Photon energy:** $E = h \nu = pc$ \hspace{1cm} [Joules]

**Momentum:** $p = \frac{h \nu}{c}$ \hspace{1cm} [kg-m/s], convertible to $[\text{eV/c}]$ by multiplying by $c/q$.

**Wavelength:** $\lambda = \frac{c}{\nu}$ \hspace{1cm} [meters]

$h = \text{Planck's constant } 6.6260755\times10^{-34} \text{ J-s}$  

$\nu = (\text{frequency of the electromagnetic wave associated with the light given off by the photon }) \hspace{1cm} [\text{Hz}]$

$c = \text{speed of light } 2.998 \times 10^8 \text{ m/s}$

PLANCK'S RADIATION LAW  

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$  

where:

$I(\lambda, T) = \text{light intensity } [\text{W/(m}^2\cdot\text{m})]$  

$\lambda = \text{wavelength } [\text{m}]$

$T = \text{temperature } (\text{K})$

$c = \text{speed of light } 2.998 \times 10^8 \text{ m/s}$

$h = \text{Planck's constant } 6.6260755\times10^{-34} \text{ J-s}$

$k = \text{Boltzmann's constant } 1.380658\times10^{-23} \text{ J/K}$

PHOTOELECTRIC EFFECT  

This is the way the book shows the formula, but it is a units nightmare.

$$\frac{1}{2} m v_{\text{max}}^2 = e V_0 = h \nu - \phi$$  

where:

$$\frac{1}{2} m v_{\text{max}}^2 = \text{energy in Joules, but convert to eV for the formula by dividing by } q.$$  

$e V_0 = \text{potential required to stop electrons from leaving the metal } [\text{V}]$

$h \nu = \text{Planck's constant } [6.6260755\times10^{-34} \text{ J-s}] \text{ multiplied by the frequency of light } [\text{Hz}]$. \hspace{1cm} This term will need to be divided by $q$ to obtain eV.

$\phi = \text{work function, minimum energy required to get an electron to leave the metal } [\text{eV}]$

INVERSE PHOTOELECTRIC EFFECT  

$$e V_0 = h \nu_{\text{min}} = \frac{hc}{\lambda_{\text{min}}}$$  

where:

$e V_0 = \text{the kinetic energy of an electron accelerated through a voltage } V_0 \hspace{1cm} [\text{eV}]$

$h \nu = \text{Planck's constant } [6.6260755\times10^{-34} \text{ J-s}] \text{ multiplied by the frequency of light } [\text{Hz}]$. \hspace{1cm} This term will need to be divided by $q$ to obtain eV.

$\lambda_{\text{min}} = \text{the minimum wavelength of light created when an electron gives up one photon of light energy } [\text{m}]$

DUANE-HUNT RULE  

$$\lambda_{\text{min}} = \frac{1.2398\times10^{-6}}{V_0}$$
ELECTRON ANGULAR MOMENTUM
from the Bohr model:

\[ L = mvr = n\hbar \]

where:

- \( L \) = angular momentum \([\text{kg-m}^2/\text{s}]\)
- \( m \) = mass \([\text{kg}]\)
- \( v \) = velocity \([\text{m/s}]\)
- \( r \) = radius \([\text{m}]\)
- \( n \) = principle quantum number
- \( \hbar \) = Planck's constant divided by \( 2\pi \) \([\text{J-s}]\)

\( \hbar = \frac{\pi}{2} \)

**a_0** BOHR RADIUS \([\text{m}]\)

The Bohr radius is the radius of the orbit of the hydrogen electron in the ground state \((n=1)\):

\[ a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} \]

\[ r_n = a_0 n^2 \]

\( a_0, \ r_n = \) Bohr radius \( 5.29177 \times 10^{-11} \text{ m} \), quantized radius \([\text{m}]\)

\( \varepsilon_0 = \) permittivity of free space \( 8.85 \times 10^{-12} \text{ F/m} \)

\( m_e = \) electron mass \( 9.1093897 \times 10^{-31} \text{ kg} \)

\( e = q = \) electron charge \([\text{c}]\)

\( n = \) principle quantum number

**IMPACT PARAMETER** \( b \)

The impact parameter \( b \) is the distance that a bombarding particle deviates from the direct-hit approach path, and is related to the angle \( \theta \) at which it will be deflected by the target particle.

\[ b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K \cot \frac{\theta}{2}} \]

\( b = \) direct path deviation \([\text{m}]\)

\( Z_1 = \) atomic number of the incident particle

\( Z_2 = \) atomic number of the target particle

\( e = q = \) electron charge \([\text{c}]\)

\( \varepsilon_0 = \) permittivity of free space \( 8.85 \times 10^{-12} \text{ F/m} \)

\( K = \) kinetic energy of the incident particle \( Z_1 \)

\( \Theta = \) angle of particle \( Z_1 \) deflection or scattering

**HEAD-ON SCATTERING**

When a particle of kinetic energy \( K \) and atomic number \( Z_1 \) is fired directly at the nucleus, it approaches to \( r_{min} \) before reversing direction. The entire kinetic energy is converted to Coulomb potential energy. Since \( r_{min} \) is measured to the center of the particles, they will just touch when \( r_{min} \) is the sum of their radii.

\[ r_{min} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 K} \]

\( r_{min} = \) particle separation (measured center to center) at the time that the bombarding particle reverses direction \([\text{m}]\)

**COMPTON EFFECT**

The scattering of a photon due to collision with a single electron results in a new wavelength \( \lambda' \) and a directional change of \( \angle \theta \) and is described by the following relation:

\[ \Delta \lambda = \lambda' - \lambda = \frac{h}{mc^2} \left( 1 - \cos \theta \right) \]

\( \Delta \lambda = \) scattered photon

\( \lambda' = \) scattered photon

\( \lambda = \) incident photon

\( p = \frac{h}{\lambda} \)

\( p = \) scattered photon

\( E = h\nu' \)

\( E = \) scattered photon

\( E = h\nu \)

\( E = \) incident photon

\( E_f = E_e \)

\( E_f = \) scattered photon

\( E_i = mc^2 \)

\( E_i = \) incident photon

The \( \phi \) relations come from the conservation of momentum:

\[ p_x = \frac{h}{\lambda'} \cos \theta + p_e \cos \phi \]

\[ p_y = \frac{h}{\lambda'} \sin \theta = p_e \sin \phi \]
RUTHERFORD SCATTERING

A particle of kinetic energy $K$ and atomic number $Z_1$, when fired at a target film of thickness $t$ and atomic number $Z_2$, will be deflected by an angle $\theta$.

\[
N(\theta) = \frac{N_i nt}{16} \left( \frac{e^2}{4\pi\varepsilon_0} \right)^2 \frac{Z_1^2 Z_2^2}{r^2 K^2 \sin^4 \left( \frac{\theta}{2} \right)}
\]

$N(\theta)$ = number of particles scattered per unit area $[m^{-2}]$
$
\theta$ = angle of particle $Z_i$ deflection or scattering
$N_i$ = total number of incident particles $[kg]$
$n$ = number of atoms per unit volume $[m^{-3}]$

where $\rho$ is density $[g/m^3]$, $N_i$ is Avogadro’s number, $N_{at}$ is the number of atoms per molecule, and $M_o$ is the gram-molecular weight $[g/mole]$.

$t$ = thickness of the target material $[m]$
$e = q = electron charge$ $[c]$
$\varepsilon_0$ = permittivity of free space $8.85 \times 10^{-12} F/m$
$Z_1$ = atomic number of the incident particle
$Z_2$ = atomic number of the target particle
$r$ = the radius at which the angle $\theta$ is measured $[m]$
$K$ = kinetic energy of the incident particle $Z_i$

PROBABILITY OF A PARTICLE SCATTERING BY AN ANGLE GREATER THAN $\theta$

\[
f = n \pi t \left( \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K} \right) \cot^2 \frac{\theta}{2} \]

$f$ = the probability (a value between 0 and 1)
$n$ = number of atoms per unit volume $[m^{-3}]$

where $\rho$ is density $[g/m^3]$, $N_i$ is Avogadro’s number, $N_{at}$ is the number of atoms per molecule, and $M_o$ is the gram-molecular weight $[g/mole]$.

$t$ = thickness of the target material $[m]$
$Z_1$ = atomic number of the incident particle
$Z_2$ = atomic number of the target particle
$e = q = electron charge$ $[c]$
$\varepsilon_0$ = permittivity of free space $8.85 \times 10^{-12} F/m$
$K$ = kinetic energy of the incident particle $Z_i$
$\theta$ = angle of particle $Z_i$ deflection or scattering

ELECTRON VELOCITY

This comes from the Bohr model and only applies to atoms and ions having a single electron.

\[
v_n = \frac{1}{4\pi\varepsilon_0 \hbar} \frac{Ze^2}{2\sqrt{\pi\varepsilon_0 m_r}}
\]

$v$ = electron velocity $[m/s]$
$Z$ = atomic number or number of protons in the nucleus
$e = q = electron charge$ $[c]$
$n$ = the electron orbit or shell
$\varepsilon_0$ = permittivity of free space $8.85 \times 10^{-12} F/m$
$m_r$ = mass of an electron $9.1093897 \times 10^{-31} kg$
$\hbar$ = Planck’s constant divided by $2\pi$ $[J-s]$
$r$ = the radius of the electron’s orbit $[m]$

ELECTRON ORBIT RADIUS

This comes from the Bohr model and only applies to atoms and ions having a single electron.

\[
r_n = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e Z e^2}
\]

$r_n$ = electron orbit radius in the $n$ shell $[m]$

other variables are previously defined

$\alpha_r$ RADIAL ACCELERATION

$\alpha_r$ = the radial acceleration of an orbiting electron $[m/s^2]$
$v$ = tangential velocity of the electron $[m/s]$
$r$ = electron orbit radius $[m]$

\[
\alpha_r = \frac{v^2}{r}
\]
**RYDBERG CONSTANT**

R\(_\infty\) is used in the Bohr model and is a close approximation assuming an infinite nuclear mass. \(R\) is the adjusted value. These values are appropriate for hydrogen and single-electron ions.

\[
R = \frac{\mu_e Z^2 e^4}{4 \pi c \hbar^3 (4\pi\varepsilon_0)^2}
\]

where \(\mu_e = \frac{m_e M}{m_e + M}\)

\(R_\infty\) = Rydberg constant \(1.09678 \times 10^7 \text{ m}^{-1}\) (\(1.096776 \times 10^7 \text{ m}^{-1}\) for hydrogen)

\(\mu_e\) = adjusted electron mass

\(Z\) = atomic number, or number of protons in the nucleus

\(\varepsilon_0\) = permittivity of free space \(8.85 \times 10^{-12} \text{ F/m}\)

\(c\) = speed of light \(2.998 \times 10^8 \text{ m/s}\)

\(\hbar\) = Planck’s constant divided by \(2\pi\) \([\text{J-s}]\)

\(m_e\) = mass of an electron \(9.1093897 \times 10^{-31} \text{ kg}\)

\(M\) = mass of the nucleus (essentially the same as the mass of the atom \(\Rightarrow\) atomic number \(\times 1.6605 \times 10^{-27} \text{ kg}\))

---

**MOSELEY'S EQUATION**

British physicist, Henry Moseley determined this equation experimentally for the frequency of \(L_\alpha\) x-rays. \(L_\alpha\) waves are produced by an electron decaying from the \(n=3\) orbit to the \(n=2\) or \(L\) orbit.

\[
\nu_{L_\alpha} = \frac{5}{36} c R \left( Z - 7.4 \right)^2
\]

\(\nu\) = (nu) frequency \([\text{Hz}]\)

\(c\) = speed of light \(2.998 \times 10^8 \text{ m/s}\)

\(R\) = Adjusted Rydberg constant (see above) \([\text{m}^{-1}]\)

\(Z\) = atomic number or number of protons in the nucleus

---

**SPECTRAL LINES**

This formula gives the wavelength of light emitted when an electron in a single-electron atom or ion decays from orbit \(n_u\) to \(n_l\).

\[
\frac{1}{\lambda} = Z^2 R \left( \frac{1}{n_l^2} - \frac{1}{n_u^2} \right)
\]

\(\lambda\) = wavelength \([\text{m}]\)

\(Z\) = atomic number or number of protons in the nucleus

\(R\) = Rydberg constant \((1.096776 \times 10^7 \text{ m}^{-1}\) for hydrogen)

\(n_l\) = the lower electron orbit number

\(n_u\) = the upper electron orbit number

---

**BRAGG'S LAW**

**X-ray Scattering** - X-rays reflected from a crystal experience interference effects since rays reflecting from the interior of the material take a longer path than those reflecting from the surface. Compare to ELECTRON SCATTERING below.

\[
n \lambda = 2d \sin \theta
\]

\(n\) = order of reflection (number of lattice planes in depth)

\(\lambda\) = wavelength of the incident wave \([\text{m}]\)

\(d\) = distance between lattice planes (interatomic spacing in this case) \([\text{m}]\)

\(\theta\) = angle of incidence; the angle between the incident wave and the surface of the material

---

**ELECTRON SCATTERING**

Electrons directed into a crystalline material are scattered (reflected) at various angles depending on the arrangement of lattice planes. There is more than one set of lattice planes in a crystal. The technique can be used to explore the characteristics of a material. Compare to BRAGG'S LAW above.

\[
n \lambda = D \sin \phi
\]

\(n\) = order of reflection (number of lattice planes in depth)

\(\lambda\) = wavelength of the incident wave \([\text{m}]\)

\(D\) = interatomic spacing \([\text{m}]\)

\(d\) = distance between lattice planes \([\text{m}]\)

\(\phi\) = angle between the incident and reflected waves

---

**K CLASSICAL KINETIC ENERGY**

Two expressions for kinetic energy:

\[
\frac{p^2}{2m} = K = \frac{3}{2} kT
\]

lead to a momentum-temperature relation for particles:

\[
p^2 = 3m kT
\]

\(p\) = momentum \([\text{kg-m/s}]\)

\(m\) = particle mass \([\text{kg}]\)

\(K\) = kinetic energy \([\text{J}]\)

\(k\) = Boltzmann's constant \(1.380658 \times 10^{-23} \text{ J/K}\)

\(T\) = temperature in Kelvin \((273.15\text{K} = 0\text{°C}, \Delta K = \Delta C)\)

(see page 5 for RELATIVISTIC KINETIC ENERGY)
**WAVES**

**Ψ WAVE FUNCTIONS**

Classical Wave Equation  
\[ \frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \]

We did not use this equation:

\[ \frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \]

This wave function fits the classical form, but is not a solution to the Schröedinger equation:

\[ \Psi(x, t) = A \sin(kx - \omega t + \phi) \]

More general wave functions which are solutions to the Schröedinger equation are:

\[ \Psi(x, t) = A e^{i(kx - \omega t)} = A[\cos(kx - \omega t) + i \sin(kx - \omega t)] \]

The negative sign denotes wave motion in the positive x direction, assuming omega is positive.

More general wave functions which are solutions to the Schröedinger equation are:

\[ \Psi(x, t) = A e^{i(kx - \omega t)} = A[\cos(kx - \omega t) + i \sin(kx - \omega t)] \]

The negative sign denotes wave motion in the positive x direction, assuming omega is positive.

**k WAVE NUMBER**

A component of a wave function representing the wave density relative to distance, in units of radians per unit distance [rad/m].

\[ k = \frac{2\pi}{\lambda} \]

**ω ANGULAR FREQUENCY**

A component of a wave function representing the wave density relative to time (better known as frequency), in units of radians per second [rad/s].

\[ \omega = \frac{2\pi}{T} \]

**v_{ph} PHASE VELOCITY**

The velocity of a point on a wave, e.g. the velocity of a wave peak [m/s].

\[ v_{ph} = \frac{\lambda}{T} = \frac{\omega}{k} \]

**φ PHASE CONSTANT**

The angle by which the wave is offset from zero, i.e. the angle by which the wave's zero amplitude point is offset from \( t = 0 \). [radians or degrees].

**Ψ SUM OF TWO WAVES**

(see also WaveSummingExample.pdf)

\[ \Psi_1 + \Psi_2 = 2A \cos \left( \frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t \right) \cos \left( k_{av} x - \omega_{av} t \right) \]

\( \Delta A = \) harmonic amplitude [various units?]

\( \Delta k = \) difference in wave numbers \( k_1 - k_2 \) [rad/m]

\( k_{av} = \) average wave number \( (k_1 + k_2)/2 \) [rad/m]

\( \Delta \omega = \) difference in angular frequencies \( \omega_1 - \omega_2 \) [rad/s]

\( \omega_{av} = \) average angular frequency \( (\omega_1 + \omega_2)/2 \) [rad/s]

\( x = \) distance [m]

\( t = \) time [s]

Phase Velocity:

\[ v_{ph} = \omega_{av} / k_{av} \] [m/s] velocity of a point on a wave

Group Velocity:

\[ u_{gr} = \Delta \omega / \Delta k \] [m/s] speed of the envelope

**λ de BROGLIE WAVELENGTH**

De Broglie extended the concept of waves to all matter.

\[ \lambda = \frac{h}{p} \]

\( \lambda = \) wavelength [m]

\( h = \) Planck's constant \( 6.6260755 \times 10^{-34} \) J-s

\( p = \) momentum [kg-m/s], convertible to [eV/c] by multiplying by \( c/q \).

**WAVE UNCERTAINTIES**

This has to do with the effects of combining different waves. In order to know precisely the position of the wave packet envelope (\( \Delta x \) small), we must have a large range of wave numbers (\( \Delta k \) large). In order to know precisely when the wave is at a given point (\( \Delta t \) small), we must have a large range of frequencies (\( \Delta \omega \) large). Another result of this relationship, is that an electronic component must have a large bandwidth \( \Delta \omega \) in order for its signal to respond in a short time \( \Delta t \).

\[ \Delta k \Delta x = 2\pi \]

\[ \Delta \omega \Delta t = 2\pi \]

\( \Delta k = \) the range of wave numbers, see WAVE NUMBER

\( \Delta x = \) the width of the wave envelope

\( \Delta \omega = \) the range of wave frequencies

\( \Delta t = \) a time interval
SCHRÖDINGER'S WAVE EQUATION

time-dependent form:
\[ \frac{K}{m} + U = E \]
\[-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t} \]

time-independent form:
\[-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \]
\[-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} = E - V(x) \]

\( \hbar = \text{Planck's constant divided by } 2\pi \ [\text{J-s}] \)
\( \Psi(x,t) = \text{wave function} \)
\( V = \text{voltage; can be a function of space and time } (x,t) \)
\( m = \text{mass } [\text{kg}] \)

Two separate solutions to the time-independent equation have the form:
\[ A e^{ikx} + Be^{-ikx} \]
where \( k = \sqrt{2m(E - V) / \hbar} \)

\[ A \sin(kx) + B \cos(kx) \]

Note that the wave number \( k \) is consistent in both solutions, but that the constants \( A \) and \( B \) are not consistent from one solution to the other. The values of constants \( A \) and \( B \) will be determined from boundary conditions and will also depend on which solution is chosen.

PROBABILITY

A probability is a value from zero to one. The probability may be found by the following steps:

Multiply the function by its complex conjugate and take the integral from negative infinity to positive infinity with respect to the variable in question, multiply all this by the square of a constant \( c \) and set equal to one.

\[ c^2 \int_{-\infty}^{\infty} F^* F \ dx = 1 \]

Solve for the probability constant \( c \).

The probability from \( x_1 \) to \( x_2 \) is: \( P = c^2 \int_{x_1}^{x_2} F^* F \ dx \)
SIMPLE HARMONIC MOTION

Examples of simple harmonic motion include a mass on a spring and a pendulum. The average potential energy equals the average kinetic energy equals half of the total energy. In simple harmonic motion, \( k \) is the spring constant, not the wave number.

spring constant \( k \): \( \omega = \sqrt{\frac{k}{m}} \)  
force: \( F = kx \)

potential energy \( V \): \( V = \frac{1}{2} kx^2 \)

Schrödinger Wave Equation for simple harmonic motion: \( \frac{d^2\psi}{dx^2} = \left( \alpha^2 x^2 - \beta \right) \psi \)

where \( \alpha^2 = \frac{mk}{\hbar^2} \) and \( \beta = \frac{2mE}{\hbar^2} \)

The wave equation solutions are:

\[
\psi_n = H_n(x) e^{-\alpha x^2/2}
\]

where \( H_n(x) \) are polynomials of order \( n \), where \( n = 0, 1, 2, \ldots \) and \( x \) is the variable taken to the power of \( n \). The functions \( H_n(x) \) are related by a constant to the Hermite polynomial functions.

\[
\psi_0 = \left( \frac{\alpha}{\pi} \right)^{1/4} e^{-\alpha x^2/2} \\
\psi_1 = \left( \frac{\alpha}{\pi} \right)^{1/4} \sqrt{2\alpha} x e^{-\alpha x^2/2} \\
\psi_2 = \left( \frac{\alpha}{\pi} \right)^{1/4} \sqrt{2} (2\alpha x^2 - 1) e^{-\alpha x^2/2} \\
\psi_3 = \left( \frac{\alpha}{\pi} \right)^{1/4} \sqrt{3} (x\sqrt{\alpha}) (2\alpha x^2 - 3) e^{-\alpha x^2/2}
\]

… and they call this simple!

quantized energy levels:

\[
E_n = \left( n + \frac{1}{2} \right) \hbar \omega
\]

The zero-point energy, or Heisenberg limit is the minimum energy allowed by the uncertainty principle; the energy at \( n=0 \):

\[
E_0 = \frac{1}{2} \hbar \omega
\]

---

HEISENBERG UNCERTAINTY PRINCIPLE

These relations apply to Gaussian wave packets. They describe the limits in determining the factors below.

\[
\Delta p_x \Delta x \geq \frac{\hbar}{2} \quad \Delta E \Delta t \geq \frac{\hbar}{2}
\]

\( \Delta p_x \) = the uncertainty in the momentum along the \( x \)-axis
\( \Delta x \) = the uncertainty of location along the \( x \)-axis
\( \Delta E \) = the uncertainty of the energy
\( \Delta t \) = the uncertainty of time. This also happens to be the particle lifetime. Particles you can measure the mass of (\( E=mc^2 \)) have a long lifetime.

---

INFINITE SQUARE-WELL POTENTIAL or “Particle in a Box”

This is a concept that applies to many physical situations. Consider a two-dimensional box in which a particle may be trapped by an infinite voltage potential on either side. The problem is an application of the Schrödinger Wave Equation.

The particle may have various energies represented by waves that must have an amplitude of zero at each boundary 0 and L. Thus, the energies are quantized. The probability of the particle’s location is also expressed by a wave function with zero values at the boundaries.

Wave function: \( \psi_n(x) = A \sin \left( \frac{n\pi x}{L} \right) \)

Energy levels: \( E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \)

Probability of a particle being found between \( x_1 \) and \( x_2 \):

\[
P = \int_{x_1}^{x_2} \psi^* \psi \ dx
\]

\( A = \frac{\sqrt{2}}{L} \) normalization constant

a useful identity: \( \sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta) \)
POTENTIAL BARRIER

When a particle of energy $E$ encounters a barrier of potential $V_0$, there is a possibility of either a reflected wave or a transmitted wave.

For $E > V_0$:
- kinetic energy: $K = E - V_0$
- wave number: $k_i = k_{III} = \sqrt{2mE / \hbar}$
- $k_{II} = \sqrt{2m(E - V_0) / \hbar}$
- incident wave: $\varphi_i = A e^{ik_i x}$
- reflected wave: $\varphi_i = B e^{-ik_i x}$
- transmitted wave: $\varphi_{III} = F e^{ik_{III} x}$
- trans. probability: $T = \left(1 + \frac{V_0^2 \sin^2 (k_{III} L)}{4E(E - V_0)}\right)^{-1}$
- reflection probability: $R = 1 - T$

For $E < V_0$: Classically, it is not possible for a particle of energy $E$ to cross a greater potential $V_0$, but there is a quantum mechanical possibility for this to happen called tunneling.
- kinetic energy: $K = V_0 - E$
- wave #, region II: $\kappa = \sqrt{2m(V_0 - E) / \hbar}$
- trans. probability: $T = \left(1 + \frac{V_0^2 \sinh^2 (\kappa L)}{4E(V_0 - E)}\right)^{-1}$
- when $\kappa L \gg 1$: $T = 16 \frac{E}{V_0} \left(1 - \frac{E}{V_0}\right) e^{-2\kappa L}$

3D INFINITE POTENTIAL BOX

Consider a three-dimensional box with zero voltage potential inside the box and infinite voltage outside. A particle trapped in the box is described by a wave function and has quantized energy levels.

Time-independent Schrödinger Wave Equation in three dimensions:

$$- \frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + V \psi = E \psi$$

Wave equation for the 3D infinite potential box:

$$\psi_{n_1n_2n_3} = A \sin \left(\frac{n_1 \pi x}{L_1}\right) \sin \left(\frac{n_2 \pi y}{L_2}\right) \sin \left(\frac{n_3 \pi z}{L_3}\right)$$

Energy levels:

$$E_{n_1n_2n_3} = \frac{\pi^2 \hbar^2}{2m} \left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right)$$

Degenerate energy levels may exist—that is, different combinations of $n$-values may produce equal energy values.
SCHRÖDINGER'S EQUATION – 3D
Spherical

spherical coordinate form:

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{2m}{\hbar^2} (E-V) \psi = 0
\]

separation of variables using: \( \psi(r, \theta, \phi) = R(r) \Phi(\theta) \Phi(\phi) \)

We can obtain a form with terms of \( g \) on one side and terms of \( R \) and \( f \) on the other. These are set equal to the constant \( m_l^2 \). \( m_l \) turns out to be an integer.

Another separation is performed for \( R \) and \( f \) and the constant is \( l(l+1) \), where \( l \) is an integer. The three equations are:

Azimuthal equation:

\[
\frac{1}{g} \frac{d^2 g}{d \phi^2} + m_l^2 = 0 \quad \Rightarrow \quad g = A e^{i m \phi}
\]

Radial equation:

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) + \frac{2m}{\hbar^2} (E-V) R - \frac{l(l+1)}{r^2} R = 0
\]

Angular Equation:

\[
\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{df}{d\theta} \right) + \left[ l(l+1) - \frac{m_l^2}{\sin^2 \theta} \right] f = 0
\]

\( m_l \) = magnetic quantum number; integers ranging from \(-l\) to \(+l\)

\( l \) = orbital angular momentum quantum number

\( \hbar \) = Planck's constant divided by \( 2\pi \) [J-s]

\( E \) = energy

\( V \) = voltage; can be a function of space and time \((x,t)\)

\( m \) = mass [kg]

NORMALIZING WAVE FUNCTIONS

To normalize a function, multiply the function by its complex conjugate and by the square of the normalization constant \( A \). Integrate the result from \(-\infty \) to \( \infty \) and set equal to 1 to find the value of \( A \). The normalized function is the original function multiplied by \( A \).

To normalize the wave function \( \Psi(x) \):

\[
\int_{-\infty}^{\infty} |A\Psi|^2 \, dx \quad \Rightarrow \quad A^2 \int_{-\infty}^{\infty} \Psi^2 \, dx = 1
\]

Where \( \Psi \) is an even function, we can simplify to:

\[
2A^2 \int_{0}^{\infty} \Psi^2 \, dx \quad \text{and find } A: \quad 2A^2 \int_{0}^{\infty} \Psi^2 \, dx = 1
\]

Some relations for definite integrals will be useful in solving this equation; see CalculusSummary.pdf page 3.

To normalize the wave function \( \Psi(r) \), where \( r \) is the radius in spherical coordinates:

\[
\int_{0}^{\infty} r^2 |A\Psi|^2 \, dr \quad \Rightarrow \quad A^2 \int_{0}^{\infty} r^2 \Psi^2 \, dr = 1
\]

Note that we integrate from 0 to \( \infty \) since \( r \) has no negative values.

To normalize the wave function \( \Psi(r, \theta, \phi) \):

\[
A^2 \int_{0}^{\infty} \int_{0}^{\pi} \int_{0}^{2\pi} |A\Psi|^2 \, r \, d\theta \, \sin \theta \, d\phi \, dr = 1
\]

Note that \( dr, d\theta, \) and \( d\phi \) are moved to the front of their respective integrals for clarity.

\[
R_n(r) \quad \text{RADIAL WAVE FUNCTIONS}
\]

for the hydrogen atom

<table>
<thead>
<tr>
<th>( n )</th>
<th>( l )</th>
<th>( R_n(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>( \frac{2}{a_0^{3/2}} e^{r/a_0} )</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>( \left( \frac{r}{a_0} \right)^{1/2} e^{-r/2a_0} )</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>( \frac{r}{a_0} e^{-r/2a_0} )</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>( \frac{1}{(a_0)^{3/2}} \left( \frac{2}{81^{3/2}} \right) \left( 27 - \frac{18}{a_0} + \frac{2r^2}{a_0^2} \right) e^{-r/3a_0} )</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>( \frac{1}{(a_0)^{3/2}} \left( \frac{4}{81\sqrt{6}} \right) \left( 6 - \frac{r}{a_0} \right) e^{-r/3a_0} )</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>( \frac{1}{(a_0)^{3/2}} \left( \frac{4}{81\sqrt{30}} \right) r^2 e^{-r/3a_0} )</td>
</tr>
</tbody>
</table>

\( a_0 = \text{Bohr radius} \ 5.29177 \times 10^{-11} \text{ m} \)
**P(r)dr RADIAL PROBABILITY**
The radial probability is a value from 0 to 1 indicating the probability of a particle occupying a certain area radially distant from the center of orbit. The value is found by integrating the right-hand side of the expression over the interval in question:

\[ P(r) \, dr = r^2 |R(r)|^2 \, dr \]

where:
- \( r = \) orbit radius
- \( R(r) = \) radial wave function, normalized to unity

**P(r) RADIAL PROBABILITY DENSITY**
The radial probability density depends only on \( n \) and \( l \).

\[ P(r) = r^2 |R(r)|^2 \]

where:
- \( r = \) orbit radius
- \( R(r) = \) radial wave function, normalized to unity

**\langle r \rangle RADIAL EXPECTATION VALUE**
average radius (radial wave function):

\[ \langle r \rangle = \int_0^\infty r \, P(r) \, dr = \int_0^\infty r^3 R(r) \, dr \]

where:
- \( P(r) = \) probability distribution function
- \( P(r) = r^2 |R(r)|^2 \, dr \)
- \( R(r) = \) radial wave function, normalized to unity

**ATOMS**

**QUANTUM NUMBERS**

- **n** = principal quantum number, shell number, may have values of 1, 2, 3, ...
- **l** = orbital angular momentum quantum number, subshell number, may have values of 0 to \( n-1 \). These values are sometimes expressed as letters: \( s, p, d, f, g, h, ... \)
- **\( m_l \)** = magnetic quantum number, may have integer values from \(-l\) to \(+l\) for each \( l \) (p251)
- **\( m_s \)** = magnetic spin quantum number, may have values of \(+\frac{1}{2}\) or \(-\frac{1}{2}\)

Then we introduce these new ones:

- **s** = intrinsic quantum number, \( s = \frac{1}{2} \) (p238)
- **j** = total angular momentum quantum number, \( j = l \pm s \)
  - but \( j \) is not less than 0. (p257)
- **\( m_j \)** = magnetic angular momentum quantum number, may have values from \(-j\) to \(+j\) (p257)

**Example, for \( n = 3 \):**

<table>
<thead>
<tr>
<th>( l )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j )</td>
<td>1/2</td>
<td>1/2</td>
<td>3/2</td>
</tr>
<tr>
<td>( m_j )</td>
<td>-3/2</td>
<td>-1/2</td>
<td>+1/2</td>
</tr>
<tr>
<td>( m_s )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**L ORBITAL ANGULAR MOMENTUM**
Classically, orbital angular momentum is \( pr \) or \( mvr \).

The orbital angular momentum \( L \) is a vector quantity. It components are as follows:

- **Magnitude:** \( L = \hbar \sqrt{j(j+1)} \)
- **Z-axis value:** \( L_z = m_j \hbar \)

The values of \( L_z \) and \( L_y \) cannot be determined exactly but obey the following relation:

\[ L^2 = L_x^2 + L_y^2 + L_z^2 \]

\( \hbar = \) Planck’s constant divided by \( 2\pi \) [J-s]

\( l = \) orbital angular momentum quantum number

\( m_l = \) magnetic quantum number; integers ranging from \(-l\) to \(+l\)

The orbital angular momentum quantum number was originally given letter values resulting from early visual observations:

<table>
<thead>
<tr>
<th><strong>s</strong></th>
<th><strong>p</strong></th>
<th><strong>d</strong></th>
<th><strong>f</strong></th>
<th><strong>g</strong></th>
<th><strong>h</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = 0 )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

**sharp, principal, diffuse, fundamental**
S  SPIN ANGULAR MOMENTUM
The spin angular momentum is (insert some illuminating explanation here).

Magnitude:
\[ |S| = \hbar \sqrt{S(S+1)} = \hbar \sqrt{3/4} \]

z component:
\[ S_z = m_s \hbar = \pm \hbar/2 \]

J TOTAL ANGULAR MOMENTUM
The vector sum of the orbital angular momentum and the spin angular momentum. This applies to 1-electron and many-electron atoms.
\[ J = L + S \]

J (the magnitude?) is an integer value from \( |L-S| \) to \( L+S \).

ALLO wed TRANSITIONS
The allowed energy level transitions for 1-electron atoms are
\[ \Delta n: \text{any} \quad \Delta l: \pm 1 \quad \Delta m_l: 0, \pm 1 \quad \Delta j: 0, \pm 1 \]

ZEEMAN SPLITTING (“ZAY· mahn”)
When a single-electron atom is under the influence of an external magnetic field (taken to be in the z-axis direction), each energy level \( n=1,2,3,\ldots \) is split into multiple levels, one for each quantum number \( m_l \).

The difference in energy is:
\[ \Delta E = \mu_B B m_l \]

\( \Delta E \) = difference in energy between two energy levels [J]
\( \mu_B \) = Bohr magneton \( 9.274078 \times 10^{-24} \) J/T
\( B \) = magnetic field [T]
\( m_l \) = magnetic quantum number; integers ranging from \(-l\) to \(+l\)

\[ \mu \] MAGNETIC MOMENT
Both the magnetic moment \( \mu \) and the orbital angular momentum \( L \) are vectors:
\[ \vec{m} = -\frac{e}{2m} \vec{L} \]
\( m \) = mass of the orbiting particle [kg]

MANY-ELECTRON ATOMS
SPECTROSCOPIC SYMBOLS
The energy state of an atom having 1 or 2 electrons in its outer shell can be represented in the form
\[ n^{2S+1}L_j \]

\( n \) = shell number
\( S \) = intrinsic spin angular momentum quantum number; \( \frac{1}{2} \) for a single-electron shell, 0 or 1 \( (S_1 + S_2) \) for the 2-electron shell
\( L \) = angular momentum quantum number; \( l \) for single-electron shell, \( L_1 + L_2 \) for a 2-electron shell, expressed as a capital letter: \( S=0, P=1, D=2, F=3, G=4, H=5, I=6 \).
\( j \) = total angular momentum quantum number \( j = l \pm s \).

I'm not sure how to tell whether it's plus or minus, but I think it has to be the lower value of \( j \) to be in the ground state. \( j \) is positive only.

ORDER OF ELECTRON FILLING
Here’s a way to remember the order in which the outer shells of atoms are filled by electrons:

Form groups of \( l \)-numbers like this. The first group is just the lowest value for \( l \): \( s \). The next value of \( l \) is \( p \); form a new group with \( s \). The third value of \( l \) is \( d \); form the third group with \( d, p, \) and \( s \). You get a list of groups like this:

\[ \begin{array}{cccccccccc}
1s & 2s & 2p & 3s & 3p & 4s & 3d & 4p & 5s & 4d & 5p & 6s & 4f & 5d & \ldots
\end{array} \]

Now, in a column, write each group twice beginning with the single \( s \) that is the first group.

Next number each \( s \) beginning with 1, placing the number in front of the \( s \). This is as far as I have gone with the list at right.

The next step is to number each \( p \) beginning with the number 2.

Then number each \( d \) beginning with the number 3.

Number each \( f \) beginning with 4, and so on.

The result will be the order of filling (there are a few exceptions) and will look like this:

1s 2s 2p 3s 3p 4s 4p 5s 5p 6s 4f 5d and so on.
**g LANDÉ g FACTOR**
A dimensionless number that helps make physics complicated. Used in ANOMALOUS ZEEMAN SPLITTING
\[ g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \]

**ALLOWED PHOTON TRANSITIONS**
The allowed photon energy level transitions for many-electron atoms are
\[ \Delta L: \pm 1 \quad \Delta J: 0, \pm 1, \text{ but } J \text{ can't transition from } 0 \text{ to } 0. \]
\[ \Delta S: 0 \quad \Delta m_J: 0, \pm 1, \text{ but can't transition from } 0 \text{ to } 0 \text{ when } \Delta J=0. \]
Other transitions are possible—just not likely.

**θ MINIMUM ANGLE BETWEEN J AND THE Z-AXIS**
There were exercises where we had to calculate this. I don't know what the significance is. This is done similarly for \( L \) and \( S \) as well.
Example: \( j = \frac{5}{2} \)
\[ \cos \theta = \frac{j \times h}{\hbar \sqrt{j(j+1)}} \]
\[ \cos^2 \theta = \frac{j^2}{j(j+1)} \]
\[ \cos \theta = \frac{j}{\sqrt{j(j+1)}} \]

**SPLITTING DUE TO SPIN**
For each state described by quantum numbers \( n, l, m_l \), there are two states defined by the magnetic spin numbers \( m_s = \pm 1/2 \). These two levels have the same energy except when the atom is influenced by an external magnetic field.
The lower of the two energy levels is aligned with the magnetic field.
\[ |\Delta E| = \frac{\hbar c}{\lambda} |\Delta \lambda| \]
\( \Delta E \) = difference in energy between two (split) energy levels \( m_s = \pm 1/2 \) [J]
\( \Delta \lambda \) = difference in wavelengths for the transitions to the ground state for each energy level [m]
\( \lambda \) = wavelength for the transitions to the ground state for the lower of the two energy levels (the greater of the two wavelengths) [m]
\( \hbar \) = Planck's constant 6.6260755×10⁻³⁴ J-s
\( c \) = speed of light 2.998 × 10⁸ m/s

**SPIN-ORBIT ENERGY SPLITTING**
Spin-orbit energy splitting is the splitting of energy levels caused by an internal magnetic field due to spin. This produces a greater \( \Delta E \) than the spin splitting described above. p265
\[ P.E. \text{ due to spin } V = -\hat{z} \cdot \mathbf{B} \]
\( z \)-component \( \mu_z = -g_s \left( \frac{e \hbar}{2m_e} \right) \)
energy level difference \( \Delta E = g_s \left( \frac{e \hbar}{m_e} \right) B \)
e = q = electron charge 1.6022×10⁻¹⁹ C
\( \hbar \) = Planck's constant divided by 2\( \pi \) [J-s]
\( j_z \) = \( z \)-component of the total angular momentum
\( \Delta E \) = difference in energy between two (split) energy levels \( m_s = \pm 1/2 \) [J]
gₕ = 2, the gyromagnetic ratio
\( m_e \) = mass of an electron 9.1093897×10⁻³¹ kg
B = internal magnetic field [T]
ANOMALOUS ZEEMAN SPLITTING

("ZAY· mahn")

In addition to the Zeeman splitting of the \( m_l \) energy levels described previously, and the spin-orbit energy splitting described above, there is a splitting of the \( m_j \) levels when an external magnetic field is present. The difference in energy between levels is:

\[
V = \mu_B B_{\text{ext}} g m_j
\]

\( V \) = difference in energy between two energy levels [J]
\( \mu_B \) = Bohr magneton \( 9.274078 \times 10^{-24} \) J/T
\( B_{\text{ext}} \) = external magnetic field [T]
\( g \) = Landé factor [no units]
\( m_j \) = magnetic angular momentum quantum number; half-integers ranging from \(-j\) to \(+j\)

---

STATISTICAL PHYSICS

\( v^*, \bar{v}, v_{rms} \) MOLECULAR SPEEDS [m/s]

Maxwell speed distribution:

\[
F (v) \, dv = 4\pi C e^{-\beta \frac{1}{2} m v^2} v^2 \, dv
\]

\( v^* \) most probable speed:

\[
v^* = \sqrt{\frac{2}{\beta m}} = \sqrt{\frac{2kT}{m}}
\]

\( \bar{v} \) mean speed:

\[
\bar{v} = \frac{4}{\sqrt{2\pi}} \sqrt{\frac{kT}{m}}
\]

\( v_{rms} \) root mean square speed:

\[
v_{rms} = \sqrt{\frac{m}{\beta}} = \sqrt{\frac{3kT}{m}}
\]

\( v \) = velocity [m/s]
\( C \) = normalization constant
\( k \) = Boltzmann's constant \( 1.380658 \times 10^{-23} \) J/K
\( T \) = temperature [K]
\( m \) = mass of the molecule [kg]
\( \beta \) = the parameter \( 1/kT \) [J⁻¹]

ENERGY DISTRIBUTION

Derived from Maxwell's speed distribution:

\[
F (E) = \frac{8\pi C}{\sqrt{2m^{3/2}}} e^{-\beta E} E^{1/2}
\]

\( F_{MB} \) MAXWELL-BOLTZMANN FACTOR

The Maxwell-Boltzmann factor is a value between 0 and 1 representing the probability that an energy level \( E \) is occupied by an electron (at temperature \( T \)). This is for classical systems, such as ideal gases. One way to determine if Maxwell-Boltzmann statistics are valid is to compare the de Broglie wavelength \( \lambda = h/p \) of a typical particle with the average interparticle spacing \( d \). If \( \lambda \ll d \) then Maxwell-Boltzmann statistics are generally acceptable.

\[
F_{MB} = A e^{-\beta E}
\]

\( d = \left( \frac{V}{N} \right)^{1/3} \)

\( A \) = normalization constant
\( \beta \) = the parameter \( 1/kT \) [J⁻¹]
\( d \) = space between atoms [m]
\( N \) = number of particles in volume \( V \).

Avogadro's number, \( 6.022 \times 10^{23} \), is the number of gas molecules in 22.4 liters, or \( 22.4 \times 10^{-3} \) m³, at 0°C and 1 atmosphere. Also, gas volume is proportional to temperature: \( V/T = V_2/T_2 \).
**F_{FD} FERMI-DIRAC DISTRIBUTION**
A value between 0 and 1 indicating the probability than an energy state is occupied by an electron. The Fermi-Dirac distribution is valid for fermions, particles with half-integer spins that obey the Pauli principle. Atoms and molecules consisting of an even number of fermions must be considered bosons when taken as a whole because their total spin will be zero or an integer.

\[
F_{FD} = \frac{1}{1 + e^{\beta(E-E_F)}}
\]

- \(B_1 = \) normalization constant
- \(\beta = \) the parameter \(1/kT \) \([J^{-1}]\)

**F_{BE} BOSE-EINSTEIN DISTRIBUTION**
The Bose-Einstein distribution is valid for bosons, particles with zero or integer spins that do no obey the Pauli principle. Photons, pions, and liquid \(^4\)He are bosons.

\[
F_{BE} = \frac{1}{B_2 e^{\beta E} - 1}
\]

- \(B_2 = \) normalization constant
- \(\beta = \) the parameter \(1/kT \) \([J^{-1}]\)

**E_F FERMI ENERGY \([eV]\)**
The Fermi energy depends on the density of electrons in the material. The Fermi-Dirac distribution is modified to include the Fermi energy:

\[
F_{FD} = \frac{1}{e^{[\beta(E-E_F)]} + 1}
\]

The relationship between the Fermi energy and the number density of particles is:

\[
E_F = \frac{\hbar^2}{8m} \left( \frac{3N}{\pi L^3} \right)^{2/3}
\]

- \(F_{FD} = \) Fermi-Dirac distribution, a value from 0 to 1 indicating the probability that an energy state is occupied
- \(\beta = \) the parameter \(1/kT \) \([J^{-1}]\)
- \(\hbar = \) Planck's constant \(6.6260755 \times 10^{-34} \text{ J-s}\)
- \(m = \) mass of the particle \([\text{kg}]\)
- \(N/L^3 = \) number density of the particles \([\text{m}^{-3}]\)

**T_F FERMI TEMPERATURE**
The Fermi temperature may be quite high, 80,000 K for copper.

\[
T_F = \frac{E_F}{k}
\]

- \(E_F = \) Fermi Energy \([eV]\)
- \(k = \) Boltzmann's constant \(1.380658 \times 10^{-23} \text{ J/K}\)

**u_F FERMI SPEED**
The Fermi speed, \(u_F = \sqrt{2E_F/m}\)

- \(E_F = \) Fermi Energy \([eV]\)
- \(m = \) mass (probably of the electron) \([\text{kg}]\)

**ENERGY STATES IN "PHASE SPACE"**
The points in the 1/8 sphere represent the energy states of a particle in a cube, see 3D INFINITE POTENTIAL BOX p13.

**Energy level at radius \(r\)**

\[
E = r^2 E_1
\]

where \(E_1 = \hbar^2 / 8mL^2\) (a constant equal to 1/3 of the ground state energy) and \(r = \sqrt{n_1^2 + n_2^2 + n_3^2}\) and \(E\) is whatever energy the problem is concerned with.

**Number of energy states** in a sphere of radius \(r\), i.e. the number of energy states there are with energy less than \(E\):

\[
N_r = 2 \left( \frac{1}{8} \right) \left( \frac{4}{3} \pi r^3 \right) \quad \text{or} \quad N_r = \frac{\pi}{3} \left( \frac{E}{E_1} \right)^{3/2}
\]

where the factor of 2 is due to spin degeneracy, and the factor of 1/8 is because the energy states only occupy 1/8 of the sphere where \(n_1, n_2, n_3\) are all positive.

**g(E) DENSITY OF ENERGY STATES**
Number of states per unit energy

\[
g(E) = \frac{\pi}{2} E_1^{-3/2} E^{3/2} \quad g(E) = \frac{\Delta N_r}{\Delta E} = \frac{dN_r}{dE}
\]

- \(N_r = \) number of energy states in a sphere of radius \(r\)
- \(E_1 = \) a constant equal to 1/3 of the ground state energy
**$n(E)$ DENSITY OF OCCUPIED STATES**
Number of occupied states per unit energy

\[
n(E) = F_{FD} \cdot g(E)
\]

at \(T=0,\ n(E) = \begin{cases} 
g(E), & \text{for } E < E_F \\
0, & \text{for } E > E_F
\end{cases}
\]

\(F_{FD}\) = Fermi-Dirac distribution, a value from 0 to 1 indicating the probability that an energy state is occupied
APPENDIX

CONSTANTS

<table>
<thead>
<tr>
<th>Constants</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avogadro's number</td>
<td>$N_A = 6.0221367 \times 10^{23}$</td>
</tr>
<tr>
<td>Bohr magneton</td>
<td>$\mu_B = \frac{q h}{2m_e} = 9.27407836 \times 10^{-24}$ J/T</td>
</tr>
<tr>
<td>Boltzmann's constant</td>
<td>$k = 1.380658 \times 10^{-23}$ J/K</td>
</tr>
<tr>
<td>Earth to Moon distance</td>
<td>$= 384 \times 10^6$ m</td>
</tr>
<tr>
<td>Elementary charge</td>
<td>$q = 1.60 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Electron mass</td>
<td>$m_e = 9.1093897 \times 10^{-31}$ kg</td>
</tr>
<tr>
<td>Neutron mass</td>
<td>$m_n = 0.51100$ MeV/c²</td>
</tr>
<tr>
<td>Proton mass</td>
<td>$m_p = 1.672631 \times 10^{-27}$ kg</td>
</tr>
<tr>
<td>Planck's constant</td>
<td>$\hbar = 6.6260755 \times 10^{-34}$ J-s</td>
</tr>
<tr>
<td>Rydberg constant</td>
<td>$R = 1.097373 \times 10^7$ m²</td>
</tr>
<tr>
<td>kT at room temperature</td>
<td>$kT = 0.0259$ eV</td>
</tr>
<tr>
<td>Speed of light</td>
<td>$c = 3.00 \times 10^8$ m/s</td>
</tr>
<tr>
<td>Speed of sound (air 0°C)</td>
<td>$v_s = 331.29$ m/s</td>
</tr>
<tr>
<td>1 Å (angstrom)</td>
<td>$10^{-10}$ cm = $10^{-8}$ M</td>
</tr>
<tr>
<td>1 µm (micron)</td>
<td>$10^{-6}$ cm</td>
</tr>
<tr>
<td>1 nm = 10 Å = 10⁻⁷ cm</td>
<td>$273.15 K = 0°C$</td>
</tr>
<tr>
<td>1 eV = 1.6 × 10⁻¹⁹ J</td>
<td>$1 W = 1 J/S = 1 VA$</td>
</tr>
<tr>
<td>1 V = 1 J/C</td>
<td>$1 N/C = 1 V/m$</td>
</tr>
<tr>
<td>1 J = 1 N·m = 1 C·V</td>
<td></td>
</tr>
</tbody>
</table>

UNITS

<table>
<thead>
<tr>
<th>Energy</th>
<th>Joules × $\frac{1}{q} = eV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>Kg × $\frac{c^2}{q} = eV/c^2$</td>
</tr>
<tr>
<td>Momentum</td>
<td>$kg \cdot m \times \frac{c}{q} = eV$</td>
</tr>
</tbody>
</table>

BINOMIAL EXPANSION

For $|x| < 1$:

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2!} x^2 \pm \frac{n(n-1)(n-2)}{3!} x^3 + \cdots$$

When $x$ is much less than 1: $(1 \pm x)^n = 1 \pm nx$

WAVELENGTH SPECTRUM

<table>
<thead>
<tr>
<th>Band</th>
<th>Meters</th>
<th>Ångstroms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longwave radio</td>
<td>1 - 100 km</td>
<td>$10^{13}$ - $10^{15}$</td>
</tr>
<tr>
<td>Standard Broadcast</td>
<td>100 - 1000 m</td>
<td>$10^{12}$ - $10^{13}$</td>
</tr>
<tr>
<td>Shortwave radio</td>
<td>10 - 100 m</td>
<td>$10^{11}$ - $10^{12}$</td>
</tr>
<tr>
<td>TV, FM</td>
<td>0.1 - 10 m</td>
<td>$10^9$ - $10^{11}$</td>
</tr>
<tr>
<td>Microwave</td>
<td>1 - 100 mm</td>
<td>$10^7$ - $10^9$</td>
</tr>
<tr>
<td>Infrared light</td>
<td>0.8 - 1000 μm</td>
<td>$8000$ - $10^7$</td>
</tr>
<tr>
<td>Visible light</td>
<td>360 - 690 nm</td>
<td>3600 - 6900</td>
</tr>
<tr>
<td>Ultraviolet</td>
<td>10 - 390 nm</td>
<td>100 - 3900</td>
</tr>
<tr>
<td>Gamma rays</td>
<td>100 - 5000 fm</td>
<td>0.05 - 100</td>
</tr>
<tr>
<td>Cosmic rays</td>
<td>&lt; 100 fm</td>
<td>&lt; 0.001</td>
</tr>
</tbody>
</table>

GREEK ALPHABET

<table>
<thead>
<tr>
<th>Greek Alphabet</th>
<th>Letter</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>α</td>
<td>alpha</td>
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<tr>
<td>B</td>
<td>β</td>
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<td>upsilon</td>
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<tr>
<td>Ξ</td>
<td>ζ</td>
<td>zeta</td>
</tr>
</tbody>
</table>
TRIG IDENTITIES
\[ i2 \sin x = e^{ix} - e^{-ix} \quad 2 \sin x = e^{ix} - e^{-ix} \]
\[ 2 \cos x = e^{ix} + e^{-ix} \quad 2 \cosh x = e^{ix} + e^{-ix} \]
\[ e^{ix} = \cos x + i \sin x \]
\[ \sin (A \pm B) = \sin A \cos B \pm \cos A \sin B \]
\[ \cos (A \pm B) = \cos A \cos B \mp \sin A \sin B \]
\[ \sin A + \sin B = 2 \sin \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right) \]
\[ \cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right) \]

COORDINATE TRANSFORMATIONS
Rectangular to Cylindrical:
To obtain: \( A(r, \theta, z) = \hat{r} A_r + \hat{\theta} A_\theta + \hat{z} A_z \)
\[ A_r = \sqrt{x^2 + y^2} \]
\[ \hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi \]
\[ \phi = \tan^{-1} \frac{y}{x} \]
\[ \hat{\theta} = -\hat{x} \sin \phi + \hat{y} \cos \phi \]
\[ z = z \quad \hat{z} = \hat{z} \]
Cylindrical to Rectangular:
To obtain: \( r(x, y, z) = r \hat{x} + y \hat{y} + z \hat{z} \)
\[ x = r \cos \phi \quad \hat{x} = \hat{r} \cos \phi - \hat{\theta} \sin \phi \]
\[ y = r \sin \phi \quad \hat{\theta} = -\hat{x} \sin \phi + \hat{y} \cos \phi \]
\[ z = z \quad \hat{z} = \hat{z} \]
Rectangular to Spherical:
To obtain: \( A(r, \theta, \phi) = \hat{r} A_r + \hat{\theta} A_\theta + \hat{\phi} A_\phi \)
\[ A_r = \sqrt{x^2 + y^2 + z^2} \]
\[ \hat{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta \]
\[ \theta = \frac{z \cos^{-1}}{\sqrt{x^2 + y^2 + z^2}} \]
\[ \hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi + \hat{z} \sin \theta \]
\[ \phi = \tan^{-1} \frac{y}{x} \quad \hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi \]
Spherical to Rectangular:
To obtain: \( r(x, y, z) = r \hat{x} + y \hat{y} + z \hat{z} \)
\[ x = r \sin \theta \cos \phi \quad \hat{x} = \hat{r} \sin \theta \cos \phi - \hat{\theta} \cos \phi \]
\[ y = r \sin \theta \sin \phi \quad \hat{\theta} = -\hat{x} \sin \phi + \hat{y} \cos \phi \]
\[ z = r \cos \theta \quad \hat{z} = \hat{x} \cos \theta - \hat{\theta} \sin \theta \]

GEOMETRY
SPHERE
Area \( A = 4 \pi r^2 \)
Volume \( V = \frac{4}{3} \pi r^3 \)
ELLIPSE
Area \( A = \pi AB \)
Circumference \( L = 2 \pi \sqrt{a^2 + b^2} \)